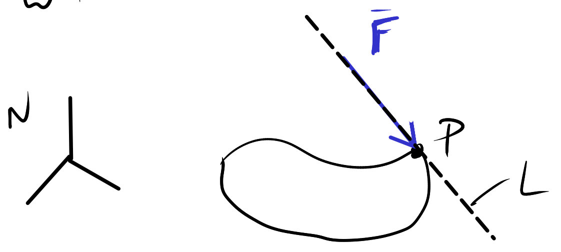
Forces, Moments, & Torques

Forces cause particles and rigid bodies to move (accelerate or angular accelerate).

Forces have direction & magnitude, so we use vectors to describe them.

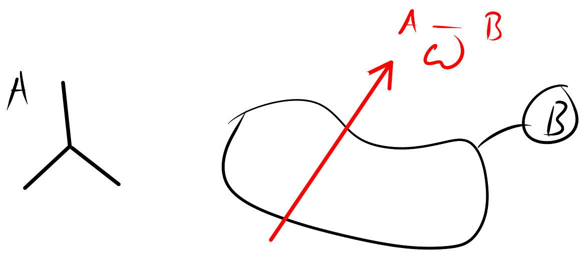
Vectors, in general, do not have a location in 3D space.

Bound vectors have direction, magnitude, and location. If a vector is bound it is associated with a line of action. If it is not associated with a line of action the vector free.



bound vector

$\bar{F}$ is bound to line of action L. L contains point P.



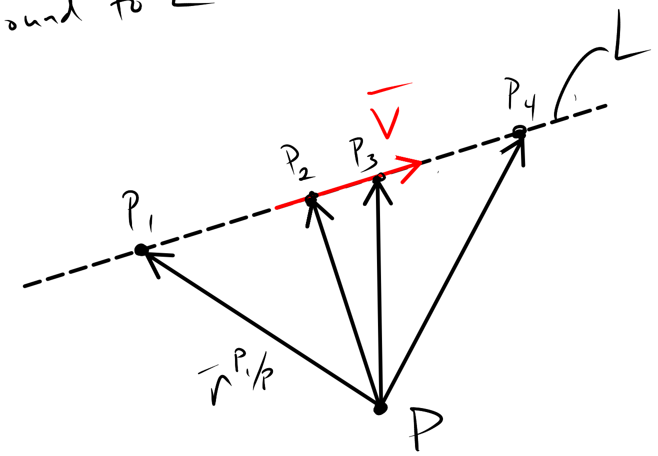
free vector

${}^A\bar{\omega}^B$ is free, same meaning no matter the location

Moment

You can calculate a moment of a bound vector with respect to a point.

$\bar{v}$ is bound to L



$$\bar{M} \triangleq \bar{r}^{P_1/P} \times \bar{v} = \bar{r}^{P_2/P} \times \bar{v} = \bar{r}^{P_3/P} \times \bar{v}$$
$$= \bar{r}^{P_4/P} \times \bar{v}$$

P_i is any point on L

If we have a set S of bound or free vectors $\bar{v}_i$ for $i=1,\dots,\nu$,
the resultant of this set is:

$$\bar{R}^{S} \triangleq \sum_{i=1}^{\nu} \bar{v}_i$$

If each vector in set S is bound we can calculate the moment of the set S with respect to a point P.

$$\bar{M}^{S/P} = \sum_{i=1}^{\nu} \bar{r}_i^{Q_i/P} \times \bar{v}_i$$

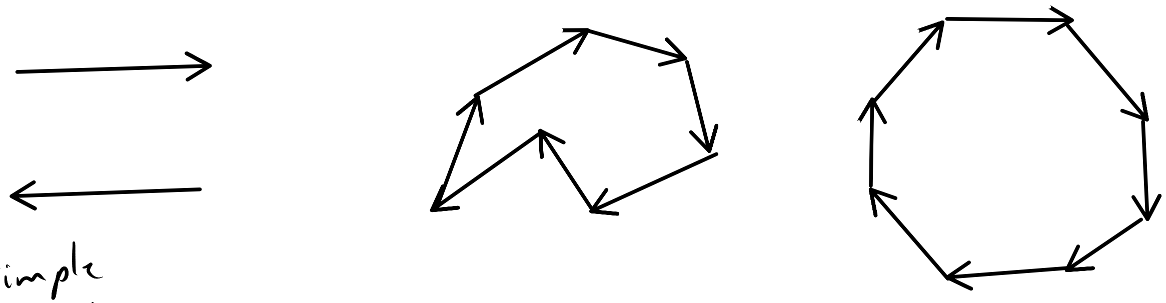
where Q_i are points on the line of action of $\bar{v}_i$.

If you know the moment of S about point Q, you can find the moment about P with this:

$$\bar{M}^{S/P} = \bar{M}^{S/Q} + \bar{r}^{Q/P} \times \bar{R}^{S/Q}$$

resultant of S bound to line of action containing Q

If the set S of bound vectors has a resultant that is zero $\bar{R}^{S}=0$, then the set S is called a **couple**.

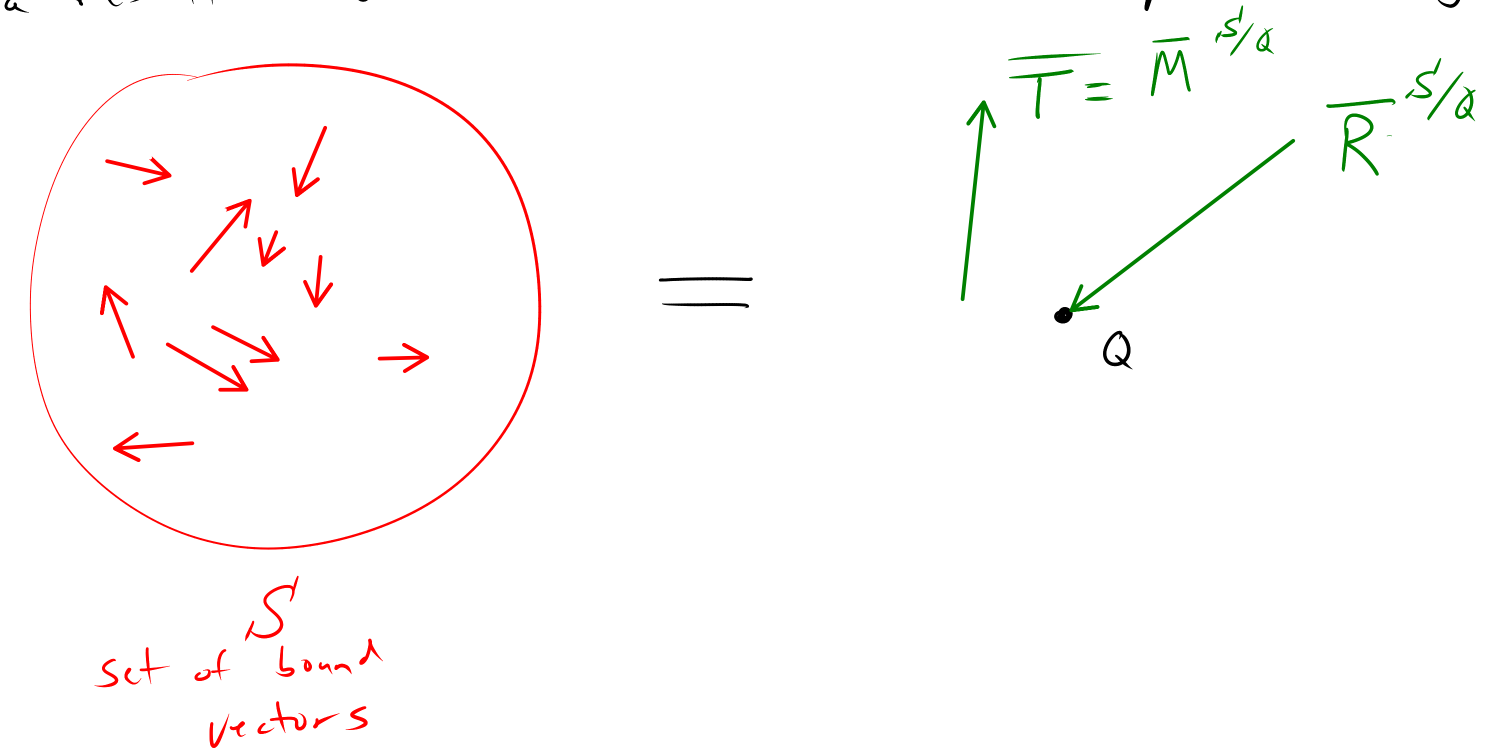


simple couple

examples of couples

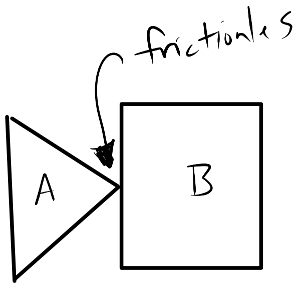
The **torque** of a couple is the moment of the couple about a point. Because $\bar{R}^{S}=0$ the torque is the same about any point.

You can replace any set $\mathcal{S}$ of bound vectors with a torque and a resultant of that $\mathcal{S}$ bound to a point of your choosing.



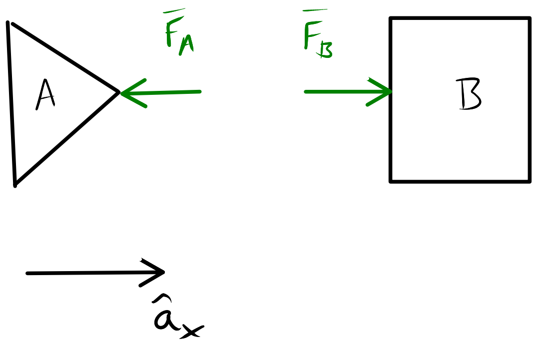
Simplest "<u>replacement</u>" of $\mathcal{S}$

Sign Conventions



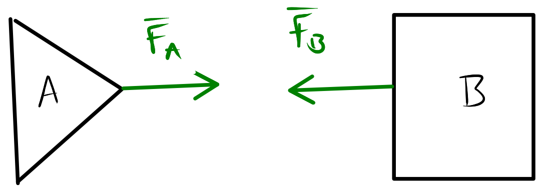
A & B contact each other

Free body diagram

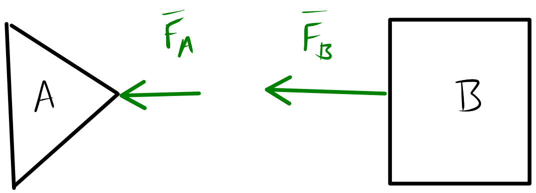


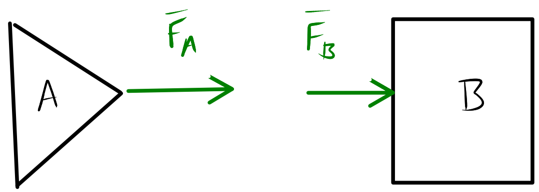
If $\bar{F}_A$ is positive then it pushes A to the left ($-\hat{a}_x$ direction).

If $\bar{F}_B$ is positive then it pushes B to the right ($\hat{a}_x$ direction)



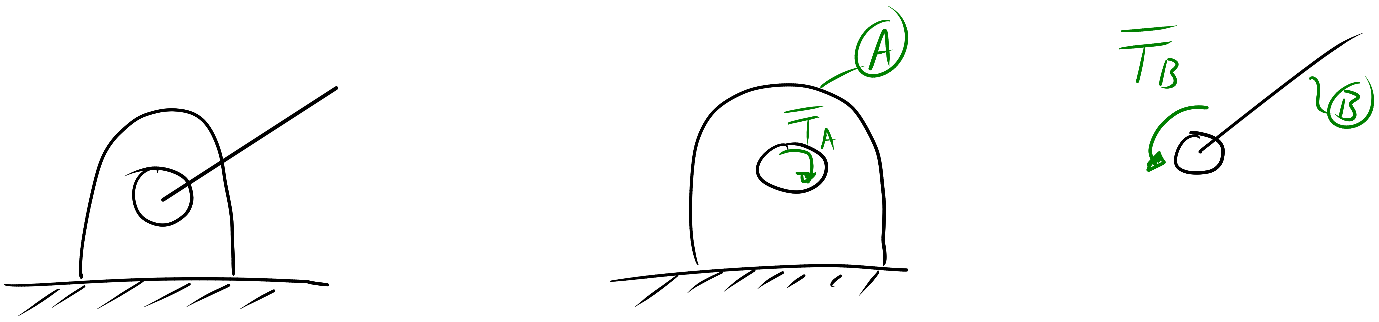
also a valid sign convention





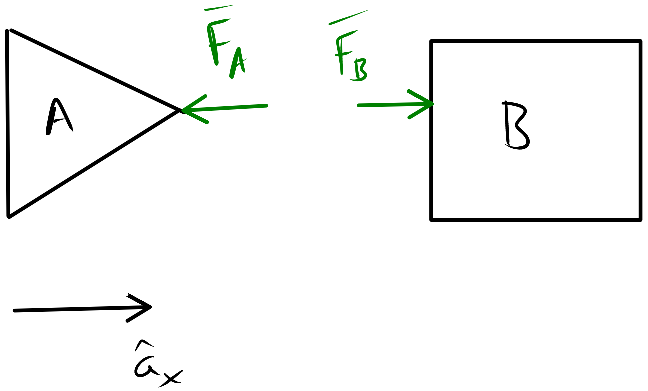
both also valid conventions!

Same for torques:



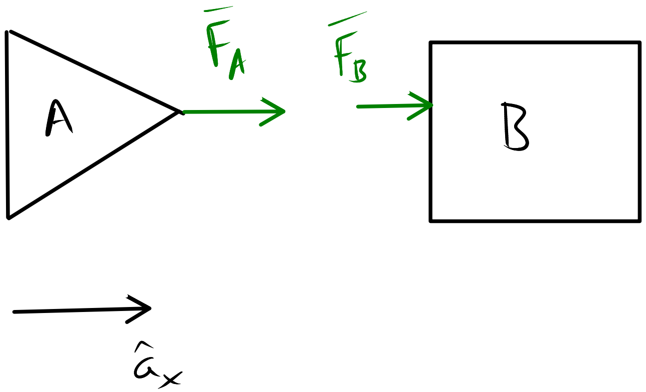
Newton's 3rd Law: Equal & Opposite

with sign convention:



then $\bar{F}_A + \bar{F}_B = 0$

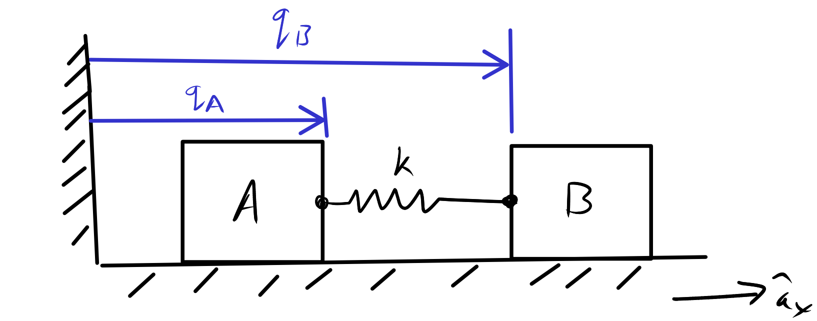
with sign convention:



then

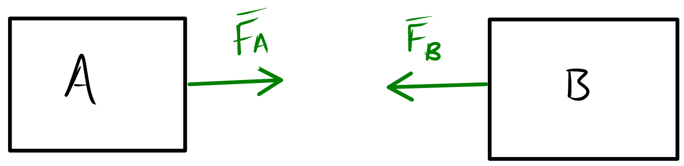
$\bar{F}_A - \bar{F}_B = 0$

Ex



$$\bar{F}_A = \Delta q k = (q_B - q_A) k \hat{a}_x$$

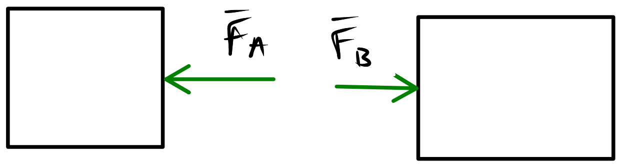
$$\bar{F}_B = \Delta q k = -(q_B - q_A) k \hat{a}_x$$



positive in tension

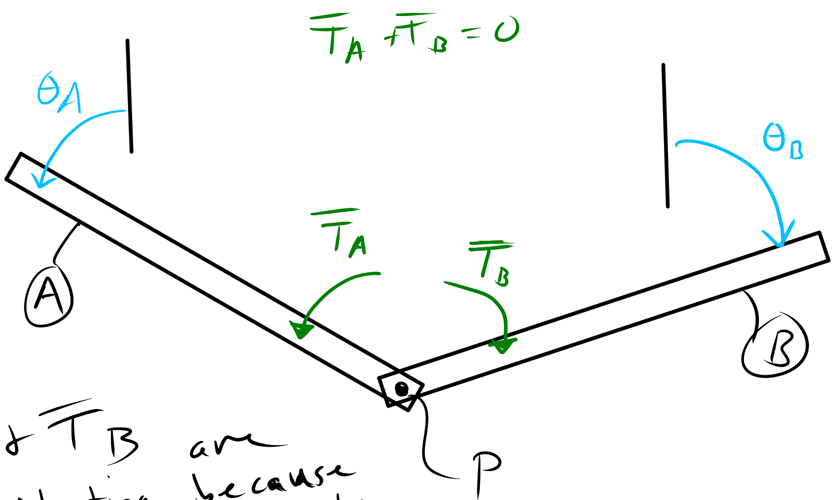
$$\bar{F}_A + \bar{F}_B = 0$$

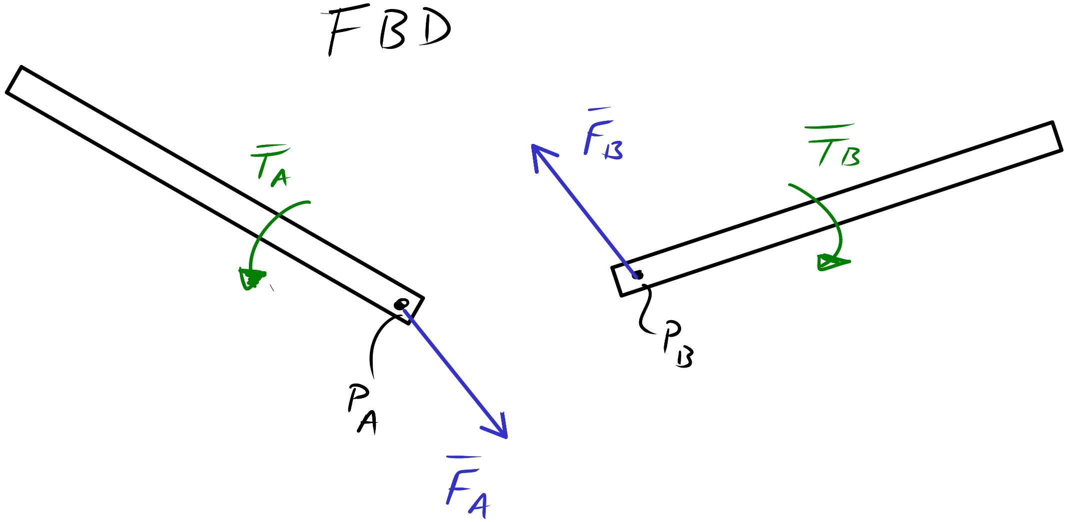
$$\bar{F}_A + \bar{F}_B = (q_B - q_A) k \hat{a}_x - (q_B - q_A) k \hat{a}_x = 0$$



positive in compression

Contributing and noncontributing forces



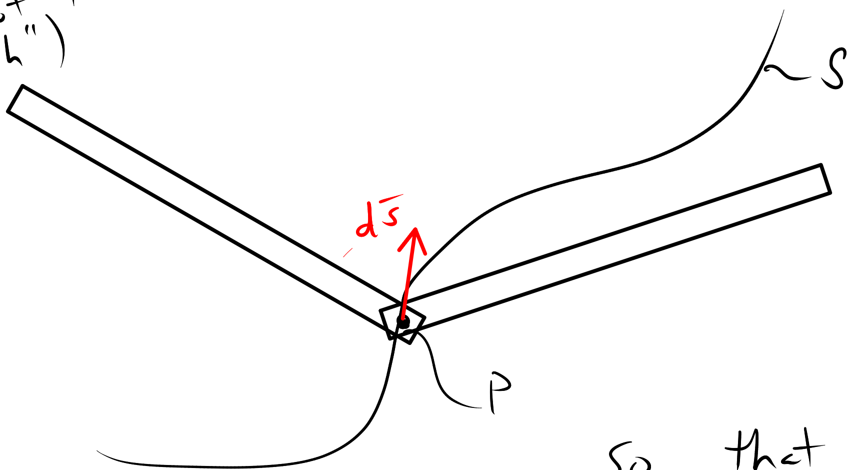


$\bar{T}_A$ & $\bar{T}_B$ are contributing because they do work on the system (θ_A and θ_B are not the same "path")

frictionless pin joint

$$W_A = \int_S \bar{F}_A \cdot d\bar{s}$$

$$W_B = \int_S \bar{F}_B \cdot d\bar{s}$$



but $\bar{F}_A + \bar{F}_B = 0$

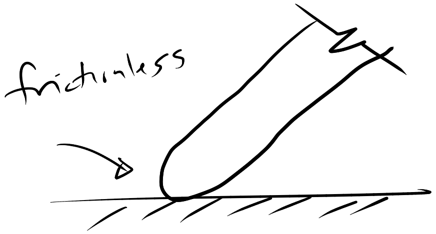
So that implies $W_A = -W_B$

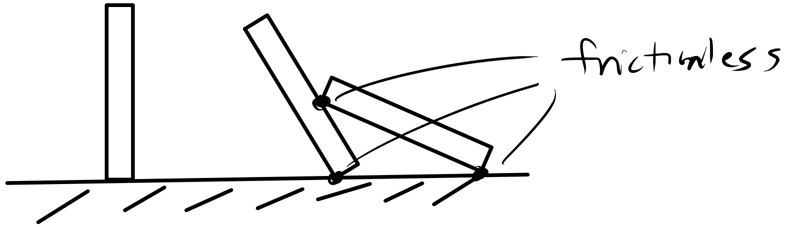
$$W = W_A + W_B = 0$$

$\bar{F}_A$ & $\bar{F}_B$ do no work on the system! → noncontributing

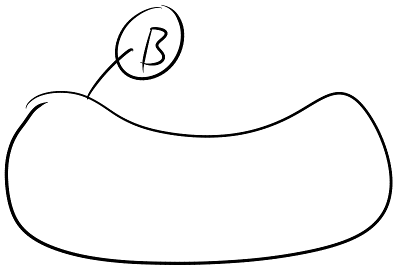
Three types of noncontributing forces:

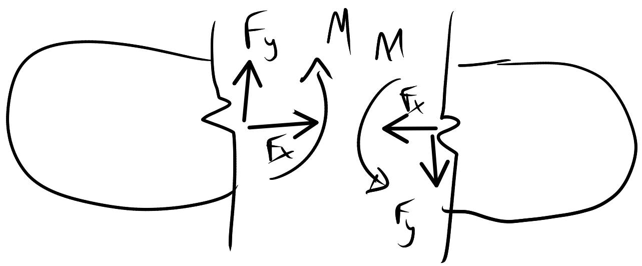
1. contact forces on particles across smooth frictionless surfaces

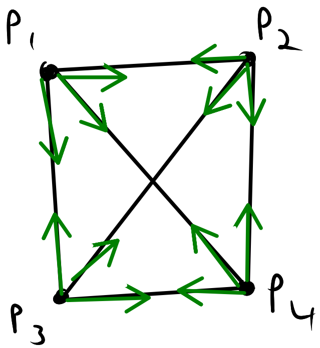


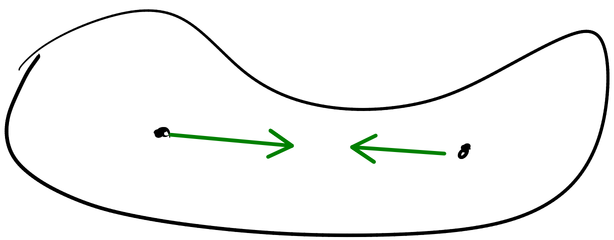


2. any internal contact and body distance forces between any two points in a rigid body

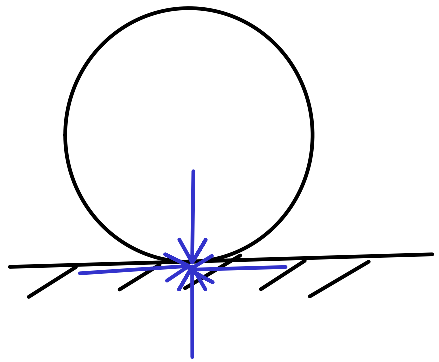








3. special case of point 1., if rolling without slipping contacts do no work



Symbolic definitions of contributing forces are possible but these may not always result in favorable numerical evaluation.

See online notes and Kane & Levinson 1985 for Ideas on implementing forces.