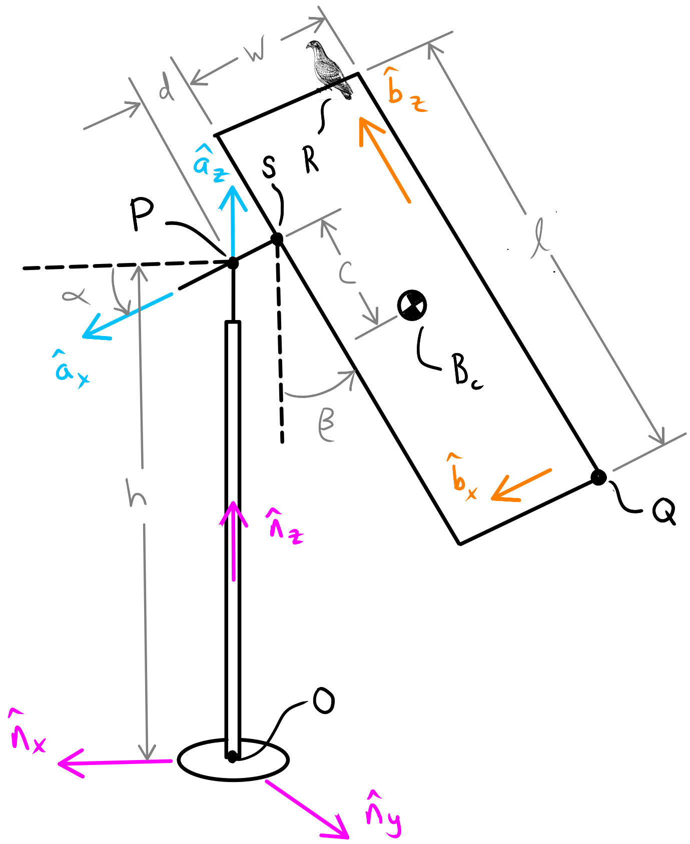


w
d
$\hat{b}_z$
$\hat{a}_z$
S
R
P
ℓ
c
α
$\hat{a}_x$
B_c
β
$\hat{b}_x$
Q
h
$\hat{n}_z$
O
$\hat{n}_x$
$\hat{n}_y$

Translational Kinematics

Translational Velocity

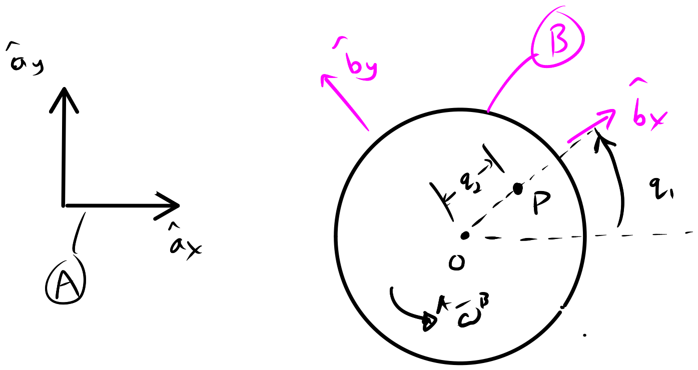
$$^{A}\bar{V}^{P} \triangleq \frac{^{A}d\,\bar{r}^{P/o}}{dt}$$

Velocity of P when the change in position is observed from reference frame A.

O is a point fixed in A.

Velocity is the time rate of change of $\bar{r}^{P/o}$ when P is not fixed in A.

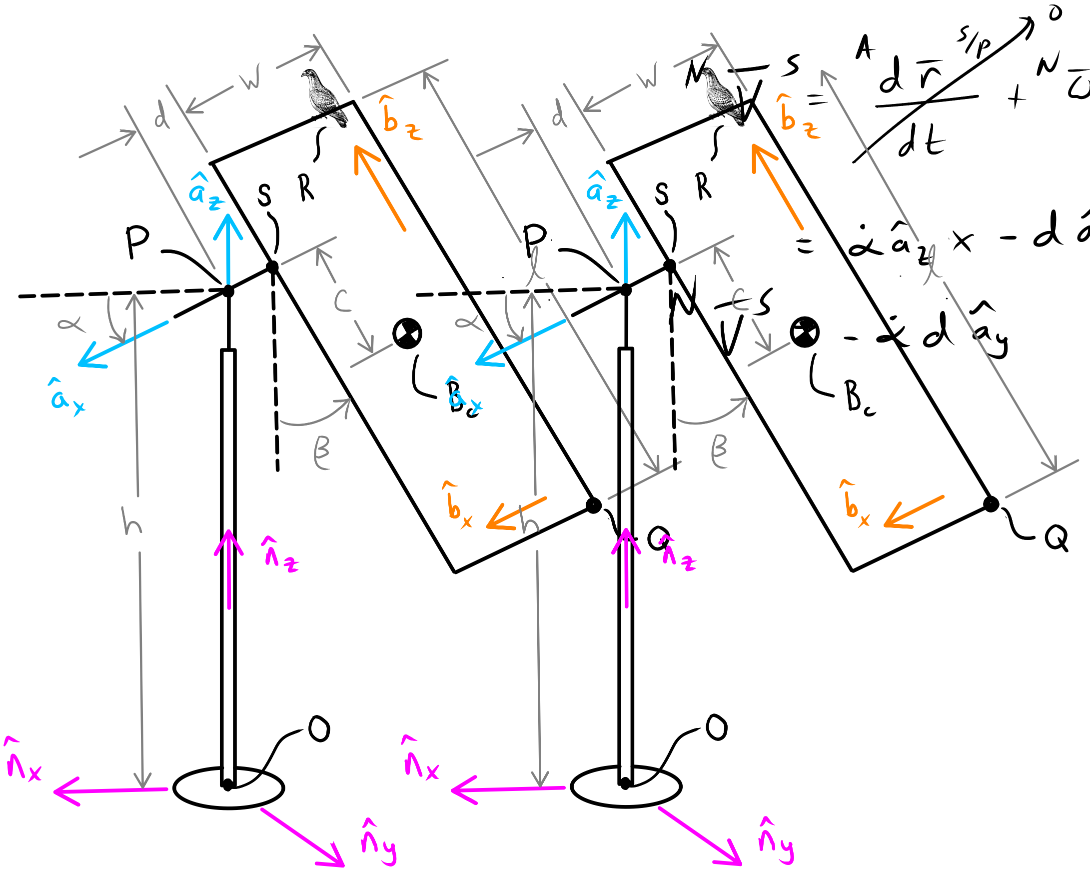
$$^{A}\bar{V}^{P} = \frac{^{B}d\,\bar{r}^{P/o}}{dt} + {}^{A}\bar{\omega}^{B} \times \bar{r}^{P/o}$$



$$\frac{^{B}d\,\bar{r}^{P/o}}{dt} = \dot{q}_2\,\hat{b}_x$$

$$^{A}\bar{\omega}^{B} \times \bar{r}^{P/o} = \dot{q}_1\,\hat{b}_z \times q_2\,\hat{b}_x$$

$$= \dot{q}_1 q_2\,\hat{b}_y$$



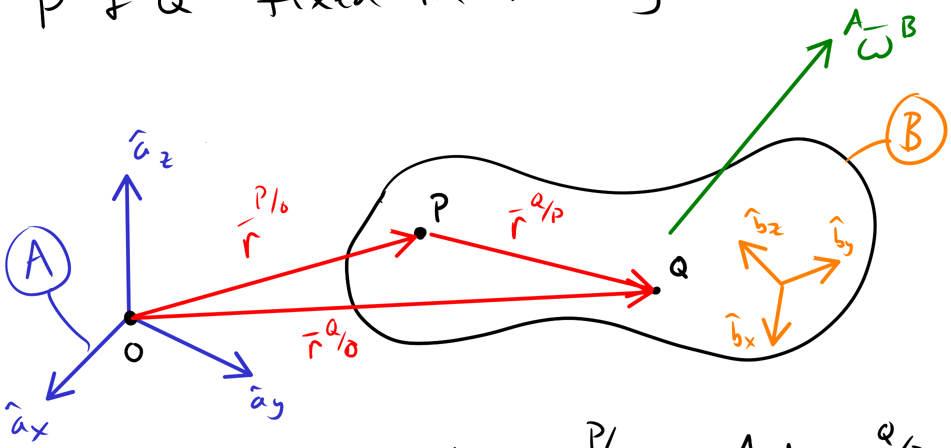
$$^{N}\bar{V}^{S} = \cancel{\frac{^{A}d\,\bar{r}^{S/P}}{dt}}^{\,0} + {}^{N}\bar{\omega}^{A} \times \bar{r}^{S/P}$$

$$= \dot{\alpha}\,\hat{a}_z \times -d\,\hat{a}_x$$

$$^{N}\bar{V}^{S} = -\dot{\alpha}\,d\,\hat{a}_y$$

Velocity Two Point Thereom

P + Q fixed in rotating reference frame B

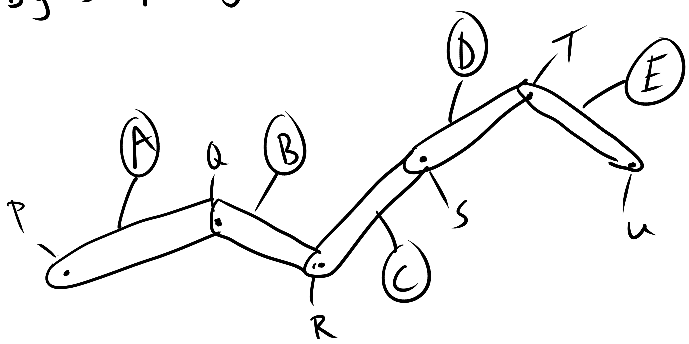


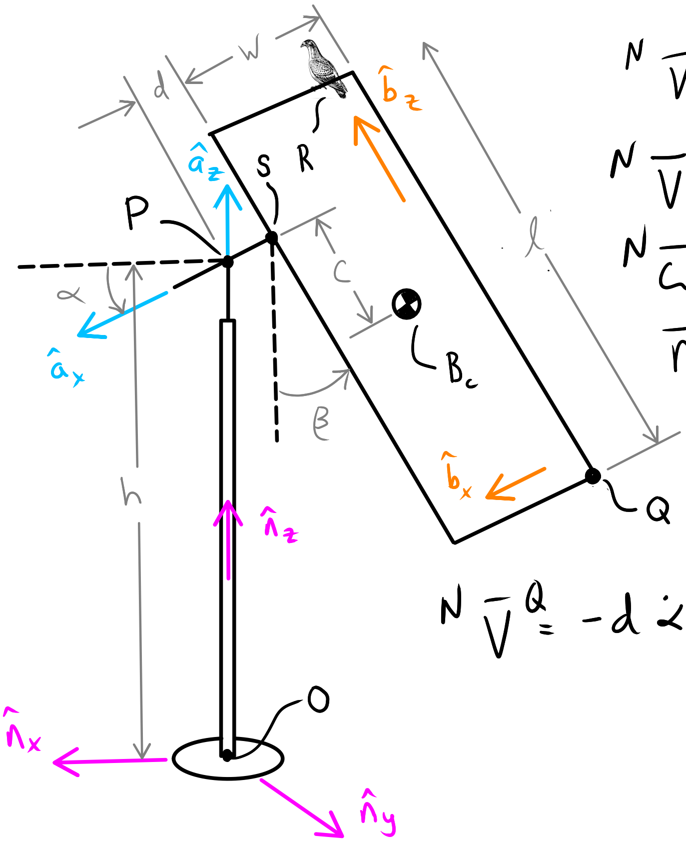
$$^A\bar{V}^Q = \frac{^A d\,\bar{r}^{Q/O}}{dt} = \frac{^A d\,\bar{r}^{P/O}}{dt} + \frac{^A d\,\bar{r}^{Q/P}}{dt}$$

$$^A\bar{V}^Q = {^A\bar{V}^P} + \cancel{\frac{^B d\,\bar{r}^{Q/P}}{dt}}^{\,0} + {^A\bar{\omega}^B} \times \bar{r}^{Q/P}$$

$$\boxed{^A\bar{V}^Q = {^A\bar{V}^P} + {^A\bar{\omega}^B} \times \bar{r}^{Q/P}}$$

Super useful for bodies connected by simple joints.





$${}^N\bar{V}^Q = {}^N\bar{V}^S + {}^N\bar{\omega}^B \times \bar{r}^{Q/S}$$

$${}^N\bar{V}^S = -d\dot{\alpha}\,\hat{a}_y$$

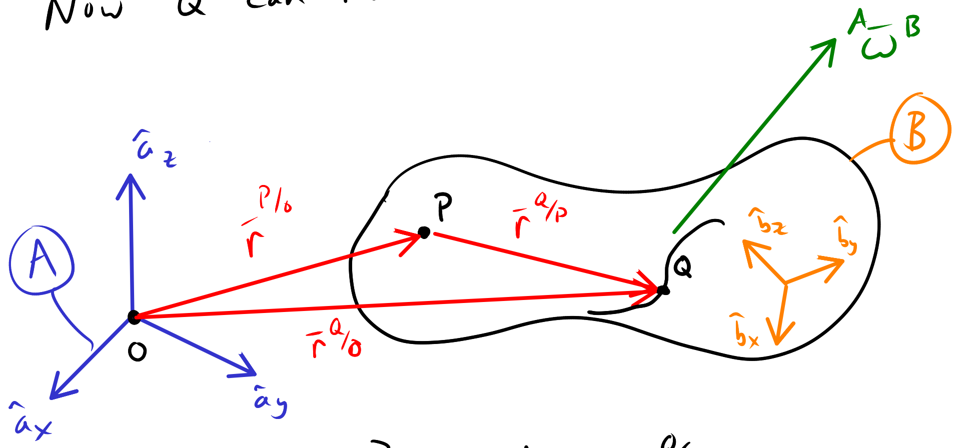
$${}^N\bar{\omega}^B = \dot{\alpha}\,\hat{a}_z + \dot{\beta}\,\hat{a}_x$$

$$\bar{r}^{Q/S} = -w\,\hat{b}_x - \left(c+\tfrac{\ell}{2}\right)\hat{b}_z$$

$${}^N\bar{V}^Q = -d\dot{\alpha}\,\hat{a}_y + (\dot{\alpha}\,\hat{a}_z + \dot{\beta}\,\hat{a}_x) \times$$
$$(-w\,\hat{b}_x - \left(c+\tfrac{\ell}{2}\right)\hat{b}_z$$

Velocity One Point Thereom

Now Q can move in reference frame B.

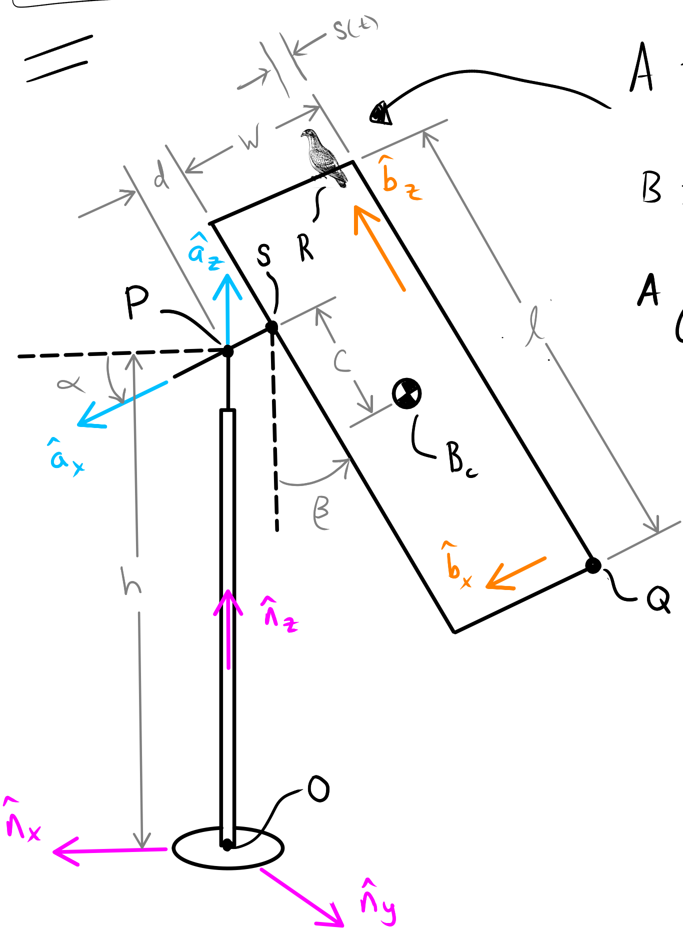


$$^A\bar V^Q = \frac{^A d\,\bar r^{P/O}}{dt} + \frac{^A d\,\bar r^{Q/P}}{dt}$$

$$= {}^A\bar V^P + \frac{^B d\,\bar r^{Q/P}}{dt} + {}^A\bar\omega^B \times \bar r^{Q/P}$$

$$\boxed{^A\bar V^Q = {}^A\bar V^P + {}^B\bar V^Q + {}^A\bar\omega^B \times \bar r^{Q/P}}$$

$=$



$$^A\bar V^R = \cancelto{0}{^A\bar V^S} + {}^B\bar V^R + {}^A\bar\omega^B \times \bar r^{R/S}$$

$$^B\bar V^R = \dot s\,\hat b_x$$

$$^A\bar\omega^B = \dot\beta\,\hat b_x$$

$$\bar r^{R/S} = -(w-s)\,\hat b_x + \left(\tfrac{\ell}{2}-c\right)\hat b_z$$

$$^A\bar V^R = \dot s\,\hat b_x + \dot\beta\,\hat b_x \times \left[(s-w)\,\hat b_x + \left(\tfrac{\ell}{2}-c\right)\hat b_z\right]$$

Acceleration of a Point

$$^{A}\bar{a}^{P} \triangleq \frac{^{A}d\,^{A}\bar{v}^{P}}{dt}$$

Acceleration Two Point Theorem

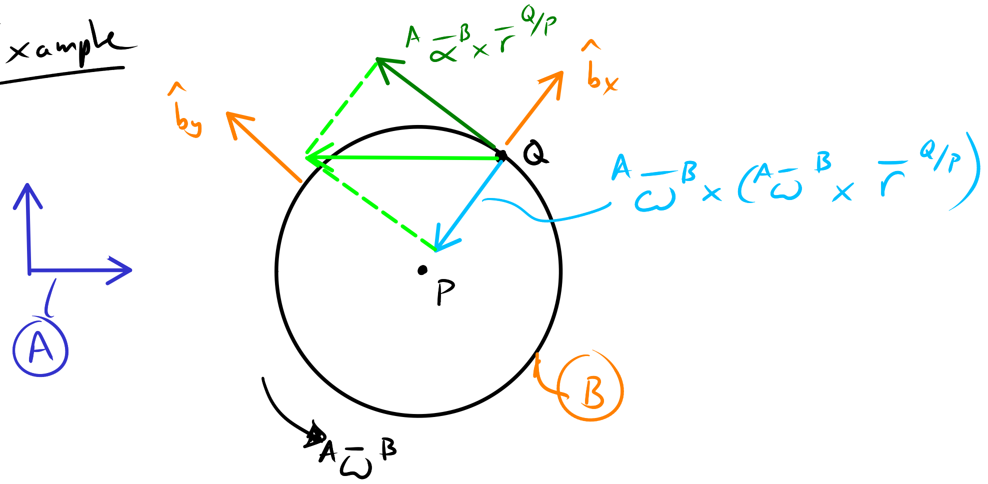
$$^{A}\bar{v}^{Q} = {^{A}\bar{v}^{P}} + {^{A}\bar{\omega}^{B}} \times \bar{r}^{Q/P}$$

Q & P are fixed in B

$$\frac{^{A}d\,^{A}\bar{v}^{Q}}{dt} = \frac{^{A}d\,^{A}\bar{v}^{P}}{dt} + \frac{^{A}d\,^{A}\bar{\omega}^{B}}{dt} \times \bar{r}^{Q/P} + {^{A}\bar{\omega}^{B}} \times \frac{^{A}d\,\bar{r}^{Q/P}}{dt}$$

$$^{A}\bar{a}^{Q} = {^{A}\bar{a}^{P}} + \underbrace{^{A}\bar{\alpha}^{B} \times \bar{r}^{Q/P}}_{\text{tangential acceleration}} + \underbrace{^{A}\bar{\omega}^{B} \times \left(^{A}\bar{\omega}^{B} \times \bar{r}^{Q/P}\right)}_{\text{centripetal acceleration}}$$

Example



Acceleration One Point Thereom

Q is moving B, P is fixed in B

$$^A\bar{V}^Q = {}^A\bar{V}^P + {}^A\bar{\omega}^B \times \bar{r}^{Q/P} + {}^B\bar{V}^Q$$

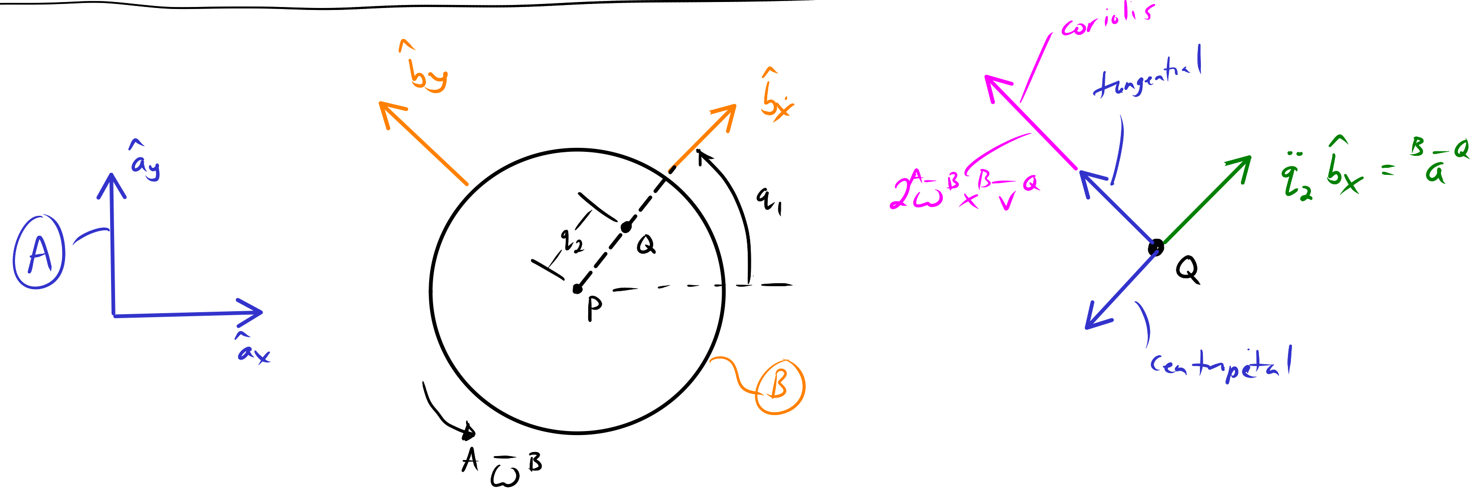
$$\frac{^A d\,{}^A\bar{V}^Q}{dt} = \frac{^A d\,{}^A\bar{V}^P}{dt} + \frac{^A d\,{}^A\bar{\omega}^B}{dt} \times \bar{r}^{Q/P} + {}^A\bar{\omega}^B \times \frac{^A d\bar{r}^{Q/P}}{dt} + \frac{^A d\,{}^B\bar{V}^Q}{dt}$$

$$^A\bar{a}^Q = {}^A\bar{a}^P + {}^A\bar{\alpha}^B \times \bar{r}^{Q/P} + {}^A\omega^B \times \left(\frac{^B d\bar{r}^{Q/P}}{dt} + {}^A\bar{\omega}^B \times \bar{r}^{Q/P} \right) + \frac{^B d\,{}^B\bar{V}^Q}{dt} + {}^A\bar{\omega}^B \times {}^B\bar{V}^Q$$

$$^A a^Q = {}^A\bar{a}^P + {}^A\bar{\alpha}^B \times \bar{r}^{Q/P} + {}^A\bar{\omega}^B \times {}^B\bar{V}^Q + {}^A\bar{\omega}^B \times ({}^A\bar{\omega}^B \times \bar{r}^{Q/P}) + {}^B a^Q + {}^A\bar{\omega}^B \times {}^B\bar{V}^Q$$

$$2\,{}^A\bar{\omega}^B \times {}^B\bar{V}^Q$$

$$\boxed{^A\bar{a}^Q = \underbrace{{}^A\bar{a}^P + {}^A\bar{\alpha}^B \times \bar{r}^{Q/P} + {}^A\bar{\omega}^B \times ({}^A\bar{\omega}^B \times \bar{r}^{Q/P})}_{\text{looks like two point thereom}} + {}^B\bar{a}^Q + \underbrace{2\,{}^A\bar{\omega}^B \times {}^B\bar{V}^Q}_{\text{Coriolis acceleration}}}$$



Kinetic sculpture pigeon

$$^A\bar{a}^R = {}^A\bar{a}^S + {}^A\bar{\alpha}^B \times \bar{r}^{R/S} + {}^A\bar{\omega}^B \times ({}^A\bar{\omega}^B \times \bar{r}^{R/S}) + 2\,{}^A\bar{\omega}^B \times {}^B\bar{V}^R + {}^B\bar{a}^R$$

Kinetic sculpture pigeon

$$^A\bar{a}^R = {}^A\bar{a}^S + {}^A\bar{\alpha}^B \times \bar{r}^{R/S} + {}^A\bar{\omega}^B \times ({}^A\bar{\omega}^B \times \bar{r}^{R/S}) + 2\,{}^A\bar{\omega}^B \times {}^B\bar{v}^R + {}^B\bar{a}^R$$