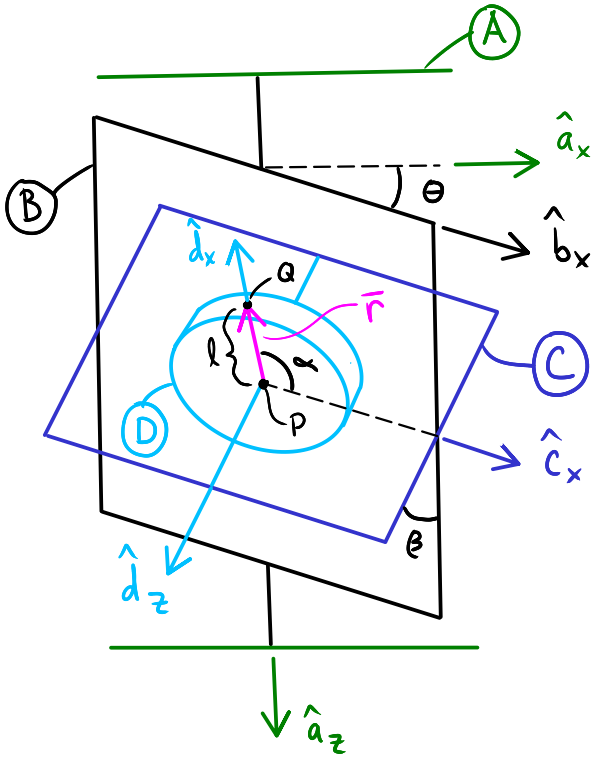

Frame	θ	β	α	l
A	✓	✓	✓	✓
B	×	✓	✓	✓
C	×	×	✓	✓
D	×	×	×	✓

Vector Differentiation

$$\bar{r} = l\,\hat{d}_x$$

$$^A\frac{\partial \bar{r}}{\partial \theta}$$ partial derivative of $\bar{r}$ when observed from A with respect to θ

$$\bar{r} = \underbrace{a_1}_{\text{x component of } \bar{r}}\hat{a}_x + \underbrace{a_2}_{\text{measure number of the z component of } \bar{r}}\hat{a}_y + a_3\hat{a}_z$$

$\hat{a}_x, \hat{a}_y, \hat{a}_z$ are fixed in A

$$^A\frac{\partial \bar{r}}{\partial \theta} = \frac{\partial a_1}{\partial \theta}\hat{a}_x + \frac{\partial a_2}{\partial \theta}\hat{a}_y + \frac{\partial a_3}{\partial \theta}\hat{a}_z$$

$\frac{\partial \bar{r}}{\partial \theta}$ is meaningless if no frame is given!

If θ, β, α, l are all themselves functions of time

$\Rightarrow \theta(t), \beta(t), \alpha(t), l(t)$ time t is the single variable

$$\bar{r}(t) = a_1(t)\,\hat{a}_x + a_2(t)\,\hat{a}_y + a_3(t)\,\hat{a}_z$$

$$^A\frac{d\bar{r}}{dt} = \frac{da_1}{dt}\hat{a}_x + \frac{da_2}{dt}\hat{a}_y + \frac{da_3}{dt}\hat{a}_z$$

Dot notation

$$^A\dot{\bar{r}} = \dot{a}_1\hat{a}_x + \dot{a}_2\hat{a}_y + \dot{a}_3\hat{a}_z$$

When the measure numbers are expressions containing functions of time, e.g. $\theta(t)$, the chain rule can be used to calculate the derivative.

$$^A\frac{d\bar{r}}{dt} = {}^A\frac{\partial\bar{r}}{\partial\theta}\frac{d\theta}{dt} + {}^A\frac{\partial\bar{r}}{\partial\beta}\frac{d\beta}{dt} + {}^A\frac{\partial\bar{r}}{\partial\alpha}\frac{d\alpha}{dt} + {}^A\frac{\partial\bar{r}}{\partial l}\frac{dl}{dt} + {}^A\frac{\partial\bar{r}}{\partial t}$$

$\left.\vphantom{\frac{1}{1}}\right\}$ if t is explicit in expressions

Second (and higher) Derivatives

$$^B\frac{\partial}{\partial y}\left({}^A\frac{\partial\bar{r}}{\partial x}\right) \neq {}^A\frac{\partial}{\partial x}\left({}^B\frac{\partial\bar{r}}{\partial y}\right)$$

$\bar{q} = t\,\hat{a}_y$ $\quad {}^B\frac{d}{dt}\left({}^A\frac{d\bar{q}}{dt}\right) = ?$

${}^A\frac{d}{dt}\left({}^B\frac{d\bar{q}}{dt}\right) = ?$

incorrectly wrote in video as c