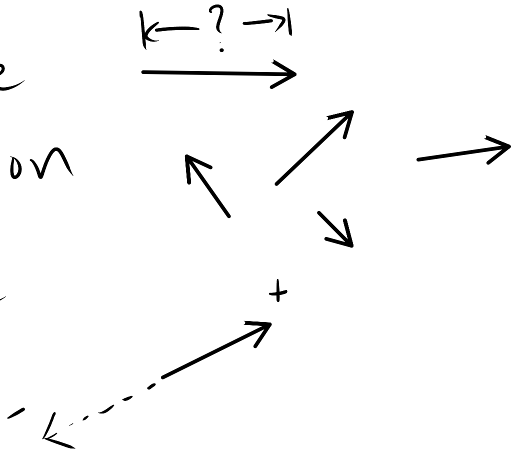
Unit Vectors

1. magnitude
2. direction
3. sense



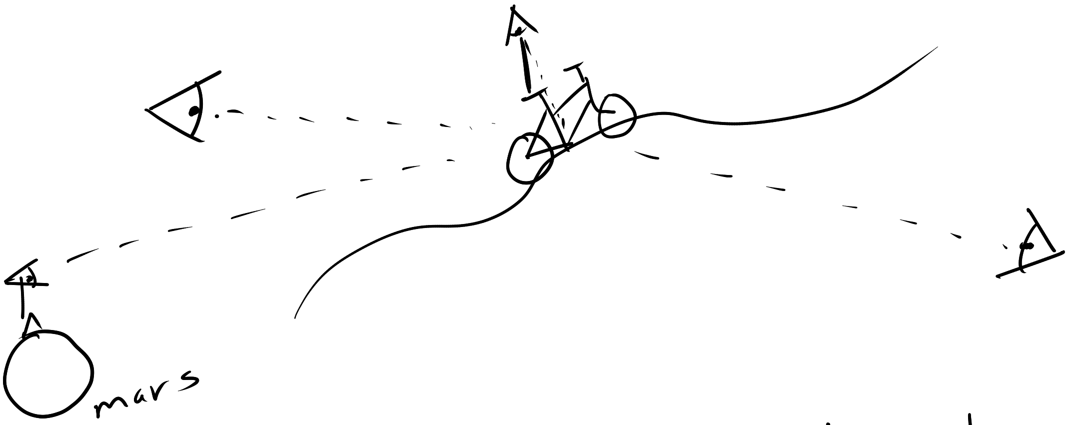
$\hat{u}$ - hat

unit vectors → magnitude = 1

Reference Frames

Euclidean 3D space : all points in some 3D space

Observer of position + motion

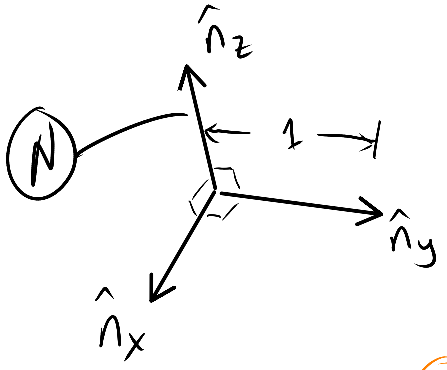


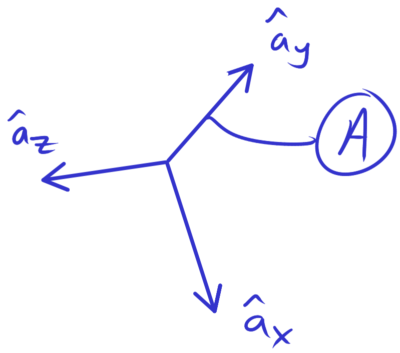
Motion is a function of the observer's orientation.

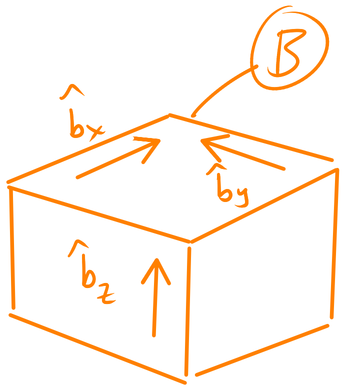
Frame of reference "reference frame" is the set of all Euclidean points fixed to the observer. An abstraction to understand motion.

We will affix 3 right-handed mutually perpendiculur unit vectors to a reference frame and use them to determine orientation among reference frames.

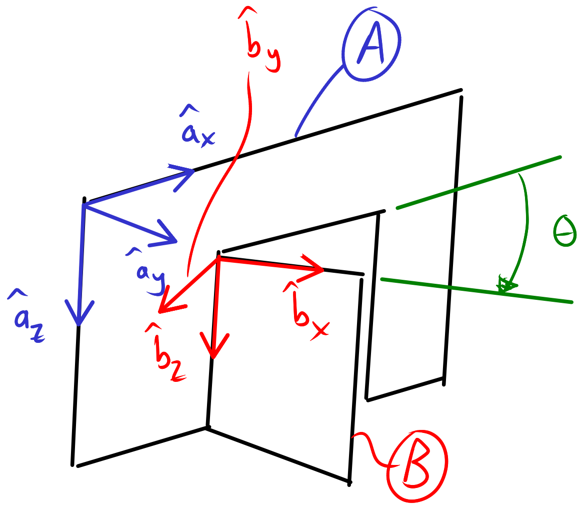
$$\hat{n}_x \times \hat{n}_y = \hat{n}_z$$





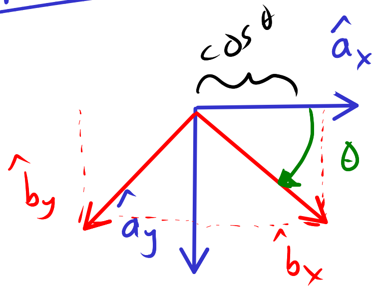


Simple 2D Orientation



$\hat{b}_z = \hat{a}_z$

Top View



$\hat{b}_x = \cos\theta\, \hat{a}_x + \sin\theta\, \hat{a}_y$ $\quad (\hat{a}_x \cdot \hat{b}_x = \cos\theta$

$\hat{b}_y = -\sin\theta\, \hat{a}_x + \cos\theta\, \hat{a}_y$

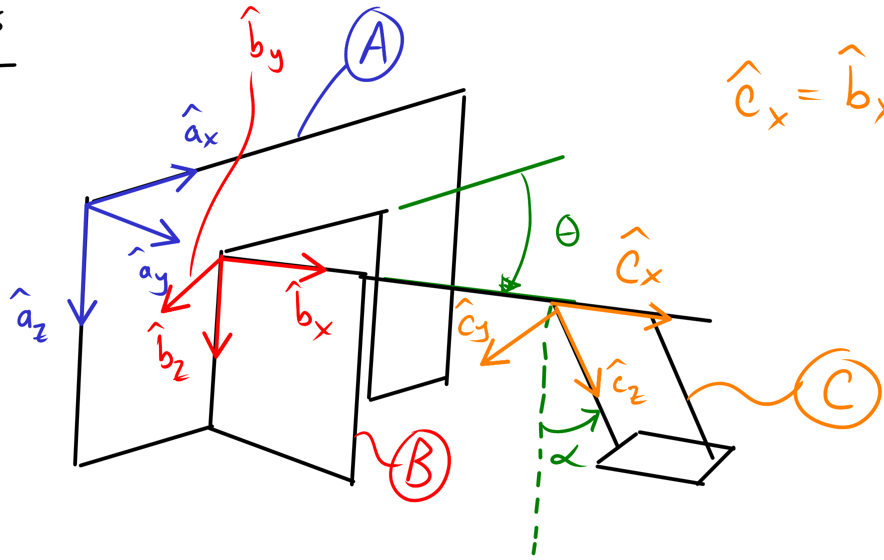
$\hat{b}_z = \hat{a}_z$

$$\begin{bmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{{}^{B}C^{A}} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

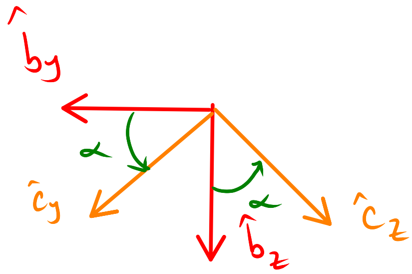
Direct cosine matrix or rotation matrix of B wrt A.

Right handed rotation of B wrt A about the shared z unit vector through the angle θ.

Successive Rotations



$\hat{c}_x = \hat{b}_x$



$$\hat{c}_y = \cos\alpha\, \hat{b}_y + \sin\alpha\, \hat{b}_z$$

$$\hat{c}_z = -\sin\alpha\, \hat{b}_y + \cos\alpha\, \hat{b}_z$$

$$\hat{c}_x = \hat{b}_x$$

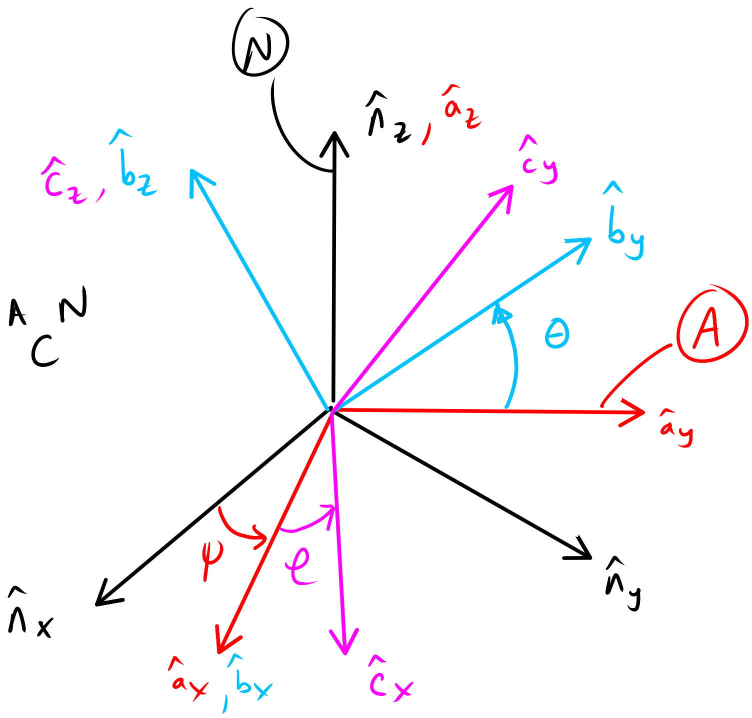
$${}^{C}C^{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

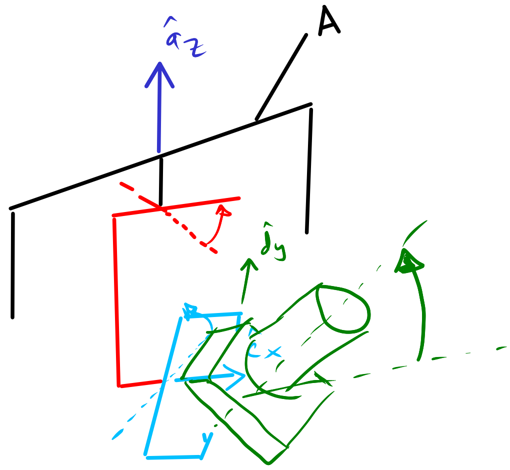
Euler Angles

3 successive simple rotations about shared axes that allows for arbitrary orientation of two reference frames with two additional or auxiliary reference frames

Z - X - Z

$${}^{C}C^{N} = {}^{C}C^{B}\,{}^{B}C^{A}\,{}^{A}C^{N}$$





$\hat{a}_z$
A
$\hat{d}_y$
x
y

$$\begin{bmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{bmatrix} = {}^{B}C^{A} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

$$\begin{bmatrix} \hat{c}_x \\ \hat{c}_y \\ \hat{c}_z \end{bmatrix} = {}^{C}C^{B} \begin{bmatrix} \hat{b}_x \\ \hat{b}_y \\ \hat{b}_z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \hat{c}_x \\ \hat{c}_y \\ \hat{c}_z \end{bmatrix} = {}^{C}C^{B}\,{}^{B}C^{A} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

$$\boxed{{}^{C}C^{A} = {}^{C}C^{B}\,{}^{B}C^{A}}$$