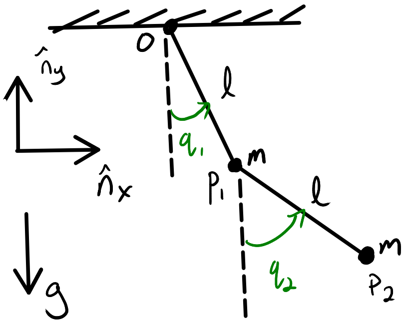
Exposing Noncontributing Forces

Formulation of Kane's Equations, eliminates all noncontributing forces, but you may want to know various distance or contact forces. The equations for any noncontributing force in a multibody system by introducing <u>auxiliary</u> <u>generalized</u> <u>speeds</u>. Forces and torques defined in the same directions of the associated auxiliary partial velocities can be used to generate extra extra equations we need.

Example: Simple Double Pendulum



$u_1 = \dot{q}_1$

$u_2 = \dot{q}_2$

$${}^N\bar{V}^{P_1} = l u_1 (c_1 \hat{n}_x + s_1 \hat{n}_y)$$

$${}^N\bar{V}^{P_2} = {}^N\bar{V}^{P_1} + l u_2 (c_2 \hat{n}_x + s_2 \hat{n}_y)$$

$${}^N\bar{a}^{P_1} = l \dot{u}_1 (c_1 \hat{n}_x + s_1 \hat{n}_y) + l u_1^2 (-s_1 \hat{n}_x + c_1 \hat{n}_y)$$

$${}^N\bar{a}^{P_2} = {}^N\bar{a}^{P_1} + l \dot{u}_2 (c_2 \hat{n}_x + s_2 \hat{n}_y) + l u_2^2 (-s_2 \hat{n}_x + c_2 \hat{n}_y)$$

$${}^N\bar{V}_1^{P_1} = l(c_1 \hat{n}_x + s_1 \hat{n}_y) \qquad {}^N\bar{V}_2^{P_1} = 0$$

$${}^N\bar{V}_1^{P_2} = l(c_1 \hat{n}_x + s_1 \hat{n}_y) \qquad {}^N\bar{V}_2^{P_2} = l(c_2 \hat{n}_x + s_2 \hat{n}_y)$$

$$\bar{R}^{P_1} = -mg\hat{n}_y \qquad \bar{R}^{*P_1} = -m\,{}^N\bar{a}^{P_1}$$

$$\bar{R}^{P_2} = -mg\hat{n}_y \qquad \bar{R}^{*P_2} = -m\,{}^N\bar{a}^{P_2}$$

$$F_1 = {}^N\bar{V}_1^{P_1}\cdot\bar{R}^{P_1} + {}^N\bar{V}_1^{P_2}\cdot\bar{R}^{P_2} = -2mgls_1$$

$$F_2 = {}^N\bar{V}_2^{P_1}\cdot\bar{R}^{P_2} + {}^N\bar{V}_2^{P_2}\cdot\bar{R}^{P_2} = -mgls_2$$

$$F_1^* = {}^N\bar{V}_1^{P_1}\cdot\bar{R}^{*P_1} + {}^N\bar{V}_1^{P_2}\cdot\bar{R}^{*P_2} =$$

$$-mlc_1(l\dot{u}_1c_1 - lu_1^2s_1) - mls_1(l\dot{u}_1s_1 + lu_1^2c_1) -$$

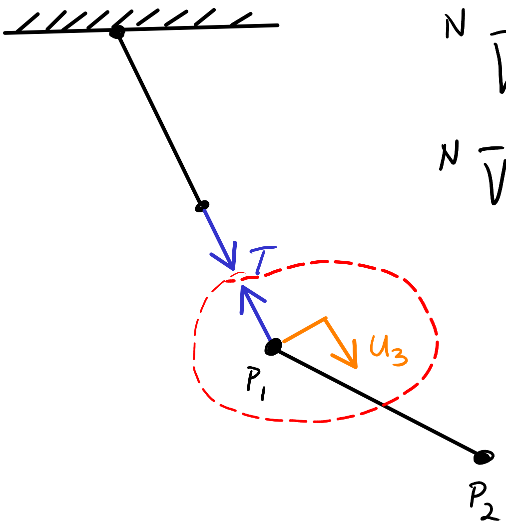
$$mlc_1(l\dot{u}_1c_1 - lu_1^2s_1 + l\dot{u}_2c_2 - lu_2^2s_2) -$$

$$mls_1(l\dot{u}_1s_1 + lu_1^2c_1 + l\dot{u}_2s_2 + lu_2^2c_2)$$

$$F_2^* = {}^N\bar{V}_2^{P_1}\cdot\bar{R}^{*P_1} + {}^N\bar{V}_2^{P_2}\cdot\bar{R}^{*P_2} =$$

$$-mlc_2(l\dot{u}_1c_1 + lu_1^2s_1 + l\dot{u}_2c_2 - lu_2^2s_2) -$$

$$mls_2(l\dot{u}_1s_1 + lu_1^2c_1 + l\dot{u}_2s_2 + lu_2^2c_2)$$



$${}^N\bar{V}_a^{P_1} = lu_1(c_1\hat{n}_x + s_1\hat{n}_y) + u_3(s_1\hat{n}_x - c_1\hat{n}_y)$$

$${}^N\bar{V}_a^{P_2} = {}^N\bar{V}_a^{P_1} + lu_2(c_2\hat{n}_x + s_2\hat{n}_y)$$

$${}^N\bar{V}_3^{P_1} = s_1\hat{n}_x - c_1\hat{n}_y$$

$${}^N\bar{V}_3^{P_2} = s_1\hat{n}_x - c_1\hat{n}_y$$

$$\bar{R}_a^{P_1} = -mg\hat{n}_y - T(s_1\hat{n}_x - c_1\hat{n}_y)$$

$$\bar{R}_a^{P_2} = -mg\hat{n}_y$$

$$F_3 = {}^N\bar{V}_3^{P_1}\cdot\bar{R}_a^{P_1} + {}^N\bar{V}_3^{P_2}\cdot\bar{R}_a^{P_2}$$

$$= -mgc_1 - T + mgc_1 = 2mgc_1 - T$$

$$F_3^* = {}^N\bar{V}_3^{P_1}\cdot\bar{R}^{*P_1} + {}^N\bar{V}_3^{P_2}\cdot\bar{R}^{*P_2}$$

$$F_3^* = lmu_1^2 + lm(s_1 s_2 + c_1 c_2)u_2^2 + lm(-s_1 c_2 + s_2 c_1)\dot{u}_2 + lmu_1^2$$

$$F_3 + F_3^* = 0 = 2gmc_1 + 2lmu_1^2 + lmu_2^2\cos(q_1 - q_2) - lm\sin(q_1 - q_2)\dot{u}_2 - T$$

$$T = 2gmc_1 + 2lmu_1^2 + lmu_2^2\cos(q_1 - q_2) - lm\sin(q_1 - q_2)\dot{u}_2$$

$$T \rightarrow f(\dot{u}_2, u_1, u_2, q_1, q_2)$$

$$\left.\begin{aligned} F_1 + F_1^* &= f(\dot{u}_1, \dot{u}_2, u_1, u_2, q_1, q_2, t) = 0 \\ F_2 + F_1^* &= f(\dot{u}_1, \dot{u}_2, u_1, u_2, q_1, q_2, t) = 0 \end{aligned}\right\} \text{independent of noncontributing forces}$$

$$F_3 + F_1^* = f(\dot{u}_1, \dot{u}_2, u_1, u_2, q_1, q_2, T, t) = 0$$

$$M_a \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ T \end{bmatrix} + \bar{g}_a = 0$$

$$M_a = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\overbrace{M_d}^{2\times 2}} & 0 \\ & & 0 \\ \hline M_a^{\dot{u}_1} & M_a^{\dot{u}_2} & M_a^{T} \end{array}\right]$$