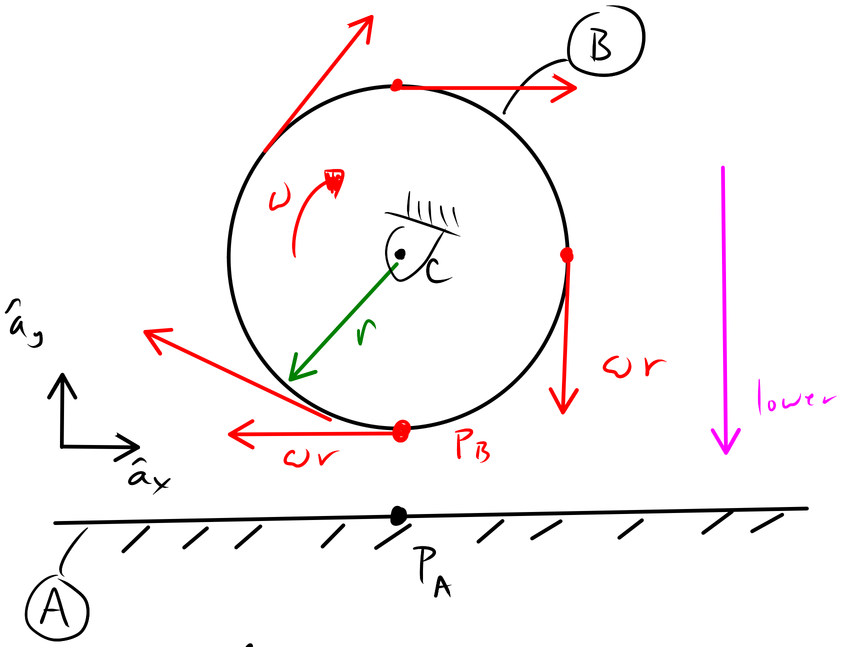
Rolling without slip

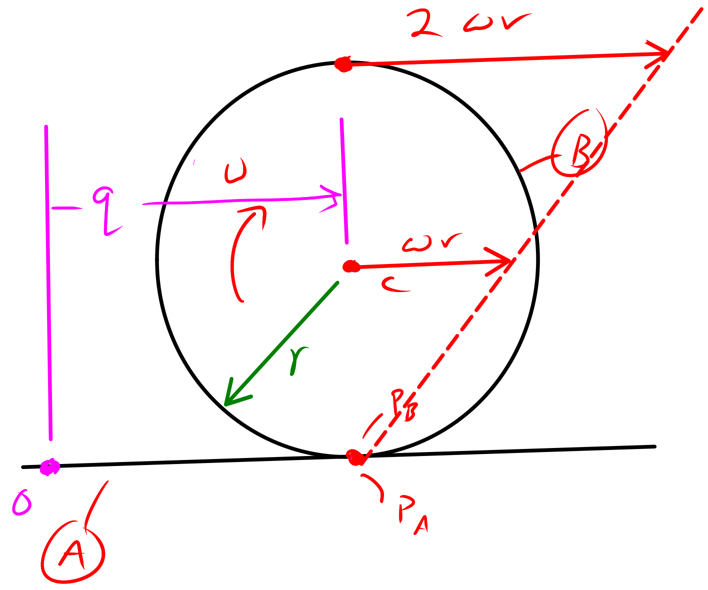


$${}^A\bar{V}^{c} = 0$$

$${}^A\bar{V}^{P_B} = -\omega r\,\hat{a}_x$$

$${}^A\bar{V}^{P_A} = 0$$

No slip



$${}^A\bar{V}^{P_B} - {}^A\bar{V}^{P_A} = 0$$

$$\dot{q} = \omega r$$

Equations of motion with nonholonomic constraints

$\bar{q} \in \mathbb{R}^n$ generalized coordinates

$\bar{u} \in \mathbb{R}^n$ generalized speeds

$$\bar{f}_k(\dot{\bar{q}}, \bar{u}, \bar{q}, t) = 0 \Rightarrow M_k \dot{\bar{q}} + \bar{g}_k = \bar{0} \quad \in \mathbb{R}^n$$

Kinematical differential equations

$$\bar{f}_d(\dot{\bar{u}}, \bar{u}, \bar{q}, t) = 0 \Rightarrow M_d \dot{\bar{u}} + \bar{g}_d = \bar{0} \quad \in \mathbb{R}^n$$

$$\bar{f}_n(\bar{u}, \bar{q}, t) = 0 \Rightarrow M_n \bar{u}_r + \bar{g}_n = \bar{0} \quad \in \mathbb{R}^m$$

algebraic equation

nonholonomic constraints

$$\bar{U} = \begin{bmatrix} \bar{u}_s \\ \bar{u}_r \end{bmatrix} \qquad \bar{u}_r \in \mathbb{R}^m \qquad \bar{u}_s \in \mathbb{R}^{p=n-m}$$

solve for dependent $\bar{u}_r$

$$\bar{u}_r = -M_n^{-1} \bar{g}_n$$

$M_n(\bar{q}, t)$

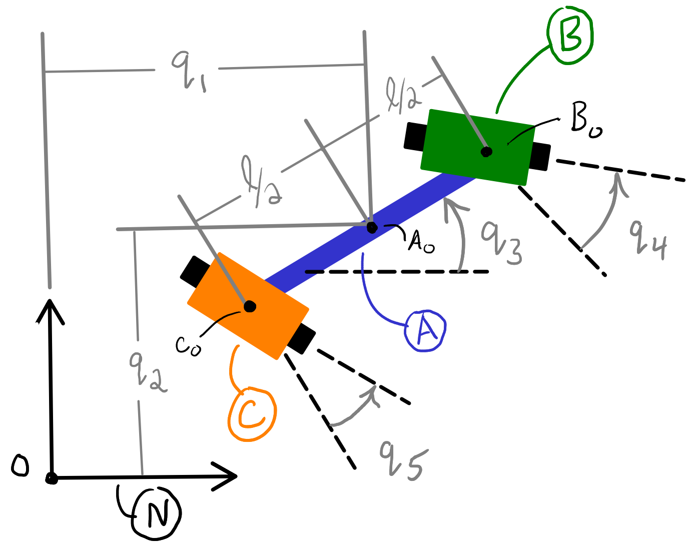
$\bar{g}_n(\bar{u}_s, \bar{q}, t)$

Substitute $\bar{u}_r$ into $\bar{f}_k$ and $\bar{f}_d$

$$\left.\begin{aligned} \bar{f}_k(\dot{\bar{q}}, \bar{u}_s, \bar{q}, t) &= \bar{0} \quad \in \mathbb{R}^n \\ \bar{f}_d(\dot{\bar{u}}_s, \bar{u}_s, \bar{q}, t) &= \bar{0} \quad \in \mathbb{R}^p \end{aligned}\right\}$$ new set of ODE where $\bar{f}_n = \bar{0}$ holds

$$\dot{\bar{q}} = -M_k^{-1} \bar{g}_k$$

$$\dot{\bar{u}}_s = -M_d^{-1} \bar{g}_d$$



B
B_o
q_1
$l/2$
$l/2$
A_o
q_3
q_4
A
c_o
q_2
C
q_5
O
N