



Equations of Motion

\bar{F}_r generalized active forces
 $\hookrightarrow \bar{F} = m\bar{a}$

\bar{F}_r^* generalized inertia forces

DoF $p=n$ (no extra ^{non +} holonomic constraints)

$$\bar{F}_r + \bar{F}_r^* = 0$$

$$\bar{F} - m\bar{a} = 0$$

dynamical differential equations, also Kane's Equations

$$\bar{F} + \bar{F}_r^* = \bar{f}_d(\dot{\bar{u}}, \bar{u}, \bar{q}, t) = 0$$

dynamical

$$\bar{f}_k(\dot{\bar{q}}, \bar{q}, t) = 0$$

kinematical

$$\bar{f}_k = M_k \dot{\bar{q}} + \bar{g}_k(\bar{u}, \bar{q}, t) = 0 \Rightarrow \dot{\bar{q}} = -M_k^{-1} \bar{g}_k \leftarrow \begin{array}{l} \text{explicit} \\ \text{in the} \\ \dot{\bar{q}} \end{array}$$

$$\bar{f}_d = M_d \dot{\bar{u}} + \bar{g}_d(\bar{u}, \bar{q}, t) = 0 \Rightarrow \dot{\bar{u}} = -M_d^{-1} \bar{g}_d \leftarrow \begin{array}{l} \text{explicit in} \\ \text{the } \dot{\bar{u}} \end{array}$$

Trajectories of $\bar{q}(t)$ & $\bar{u}(t)$

$$\bar{q}(t) = \int_{t_0}^{t_f} -M_k^{-1} \bar{g} dt$$

$$\bar{x} = \begin{bmatrix} \bar{q} \\ \bar{u} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} M_k & 0 \\ 0 & M_d \end{bmatrix}}_{2n \times 2n} \underbrace{\begin{bmatrix} \dot{\bar{x}} \\ \dot{\bar{v}} \end{bmatrix}}_{2n \times 1} + \underbrace{\begin{bmatrix} \bar{g}_k \\ \bar{g}_d \end{bmatrix}}_{2n \times 1} = \underbrace{\begin{bmatrix} \bar{0} \\ \bar{0} \end{bmatrix}}_{2n \times 1}$$

trajectories

$$M_m \dot{\bar{x}} + \bar{g}_m = 0 \Rightarrow \dot{\bar{x}} = -M_m^{-1} \bar{g}_m$$

$$\downarrow$$

$$\bar{x}(t) = \int_{t_0}^{t_f} \dot{\bar{x}} dt = \int_{t_0}^{t_f} -M_m^{-1} \bar{g}_m dt$$